

Scalar Field as Dark Matter in the Universe

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We investigate the hypothesis that the scalar field is the dark matter and the dark energy in the Cosmos, which comprises about 95% of the Universe. We show that this hypothesis explains quite well the recent observations on type Ia supernovae.

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There are really few questions in science more interesting than that of finding out which is the nature of the matter composing the Universe. It is amazing that after so much effort dedicated to such question, what is the Universe composed of?, it has not been possible to give a conclusive answer. From the latest observations, we do know that about 95% of matter in the Universe is of non baryonic nature. The old belief that matter in Cosmos is made of quarks, leptons and gauge bosons is being abandoned due to the recent observations and the inconsistencies which spring out of this assumption [1]. Now we are convinced on the existence of an exotic non baryonic sort of matter which dominates the structure of the Universe, but its nature is until now a puzzle.

Recent observations of the luminosity-redshift relation of Ia Supernovae suggest that distant galaxies are moving slower than predicted by Hubble's law, that is, an accelerated expansion of the Universe seems to hold [2,3]. Furthermore, measurements of the Cosmic Background Radiation and the mass power spectrum also suggest that the Universe has the preferable value $\Omega_0 = 1$. There should exist a kind of missing anti-gravitational matter possessing a negative pressure $p/\rho = \omega < 0$ [4] which should overcome the enormous gravitational forces between galaxies. Moreover, the interaction with the rest of the matter should be very weak to pass unnoticed at the solar system level. These observations are without doubt among the most important discoveries of the end of the last century, they gave rise to the idea that the components of the Universe are matter and vacuum energy $\Omega_0 = \Omega_M + \Omega_\Lambda$. Models such as the quintessence (a slow varying scalar field) imply $-1 < \omega < 0$ and the one using a cosmological constant, requiring $\omega = -1$, appear to be strong candidates to be such missing energy, because both of them satisfy an equation of state concerning an accelerated behavior of the Universe [5].

Observations in galaxy clusters and dynamical measurements of the mass in galaxies indicate that $\Omega_M \sim 0.4$, (see for example [6]). Observations of Ia supernovae indicate that $\Omega_\Lambda \sim 0.6$ [2,3]. These observations are in very good concordance with the preferred value $\Omega_0 \sim 1$. Everything seems to agree. Nevertheless, the matter component Ω_M decomposes itself in baryons, neutrinos, etc. and dark matter. It is observed that stars and dust (baryons) represent something like 0.3% of the whole matter of the Universe. The new measurements of the neutrino mass indicate that neutrinos contribute with about the same quantity as matter. In other words, say $\Omega_M = \Omega_b + \Omega_\nu + \dots \sim 0.05 + \Omega_{DM}$, where Ω_{DM} represents the dark matter part of the matter contributions which has a value $\Omega_{DM} \sim 0.35$. This value of the amount of baryonic matter is in concordance with the limits imposed by nucleosynthesis (see for example [1]). But we do not know the nature neither of the dark matter Ω_{DM} nor of the dark energy Ω_Λ ; we do not know what is the composition of $\Omega_{DM} + \Omega_\Lambda \sim 0.95$, i.e., the 95% of the whole matter in the Universe.

In a previous work two of us have shown that the scalar field is a strong candidate to be the dark matter in spiral galaxies [7]. Using the hypothesis that the scalar field is the dark matter in galaxies, we were able to reproduce the rotation curves profile of stars going around spiral galaxies. In fact the scalar potential arising for the explanation of rotation curves of galaxies is exponential. Moreover, by using a Monte Carlo simulation, Huterer and Turner have been able to reconstruct an exponential potential for quintessence which brings the Universe into an accelerating epoch [8]. In this last work there is no explanation for the nature of dark matter, it is taken the value $\Omega_{DM} \sim 0.35$ without further comments. Recently, there are other papers where the late time attractor solutions for the exponential potential are studied [9–11]. If

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we are consistent with our previous work, this dark matter should be also of scalar nature representing the 35% of the matter of the Universe. In this letter we show that the hypothesis that the scalar field is the dark matter and the dark energy of the Universe is consistent with Ia supernovae observations and it could imply that the scalar field is the dominant matter in the Universe, determining its structure at a cosmological and at a galactic level. In other words, in this letter we demonstrate that the hypothesis that the scalar field represents more than 95% of the matter in the Universe is consistent with the recent observations on Ia supernovae.

We assume Universe is homogenous and isotropic, so we start with the FRW metric

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2(\theta)d\phi^2) \right] \quad (1)$$

The equations governing a Universe with a scalar field Φ and a scalar potential $V(\Phi)$ are

$$\ddot{\Phi} + 3\frac{\dot{a}}{a}\dot{\Phi} + \frac{dV}{d\Phi} = 0, \quad (2)$$

$$\left(\frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} = \frac{\kappa_o}{3} (\rho + \rho_\Phi) \quad (3)$$

where $\rho_\Phi = \frac{1}{2}\dot{\Phi}^2 + V(\Phi)$ is the density of the scalar field, ρ is the density of the baryons, plus neutrinos, plus radiation, etc, and $\kappa_o = 8\pi G$. In order to write the field equations (2) and (3) in a more convenient form, we follow [12]. We define the function $F(a)$ such that $V(\Phi(a)) = F(a)/a^6$. Using the variable $d\eta = 1/a^3 dt$, we can find a first integral of the field equation (2)

$$\frac{1}{2}\dot{\Phi}^2 + V(\Phi) = \frac{6}{a^6} \int da \frac{F}{a} + \frac{C}{a^6} = \rho_\Phi \quad (4)$$

being C an integration constant. When the scale factor is considered as the independent variable, it is possible to integrate the field equations up to quadratures [12]

$$t - t_0 = \sqrt{3} \int \frac{da}{a\sqrt{\kappa_o(\rho_\Phi + \rho) - 3k/a^2}} \quad (5)$$

$$\Phi - \Phi_0 = \sqrt{6} \int \frac{da}{a} \left[\frac{\rho_\Phi - F/a^6}{\kappa_o(\rho_\Phi + \rho) - 3k/a^2} \right]^{1/2} \quad (6)$$

In order to compare the data obtained from the Ia supernovae observations with a scalar field dominated Universe, we write the magnitude-redshift relation [2]

$$m_B^{effective} = \tilde{M}_B + 5 \log D_L(z; \Omega_i, \Omega_\Phi) \quad (7)$$

where $D_L = H_0 d_L$ is the ‘‘Hubble-constant-free’’ luminosity distance and $\tilde{M}_B := M_B - 5 \log H_0 + 25$ is the

‘‘Hubble-constant-free’’ B-band absolute magnitude at the maximum of a Ia supernovae. The luminosity distance D_L depends on the model we are working with. In what follows we compare the observational measurements obtained for $m_B^{effective}$ with a theory defining a scalar field dominated Universe. Using equation (4), the luminosity distance which depends on the geometry and on the contents of the Universe in the FRW cosmology (see for example [13]), reads for our case

$$d_l(z; \Omega_i, \Omega_\Phi, H_0) = \frac{(1+z)}{H_0 \sqrt{|k|}} \text{sinn} \left(\sqrt{|k|} \int_{\frac{1}{1+z}}^1 \frac{dx}{\sqrt{U_\Phi}} \right) \quad (8)$$

where

$$U_\Phi := \left(\sum_i \Omega_i x^{(1-3w_i)} \right) - x^2(1 - \Omega_o) + \frac{1}{\rho_c x^2} \left(6 \int dx' \frac{F(x')}{x'} + C \right) \quad (9)$$

and

$$\text{sinn}(r) = \begin{cases} \sin(r) & (k = +1) \\ r & (k = 0) \\ \sinh(r) & (k = -1) \end{cases}$$

where i labels for b (baryonic), ν (neutrinos), r (radiation), etc. with equations of state $p_i = w_i \rho_i$ for each component. If we rescale $a_0 = 1$ today, then $x = a = 1/(1+z)$, being z the redshift. Let us now compare the expression (8) with the function used to fit SNe Ia measurements [14], with an equation of state $p_x = w_x \rho_x$ for the unknown energy. In this case the luminosity distance is given by the equation (8) with U_X in place of U_Φ , where

$$U_X := \left(\sum_i \Omega_i x^{(1-3w_i)} \right) - x^2(1 - \Omega_o) + x^{(1-3w_x)} \Omega_x. \quad (10)$$

Observe that both expressions (9) and (10) are very similar, the only differences are the integral term and the one containing the constant C . Thus, this comparison extremely suggest that $C = 0$ and $F(x) = V_0 x^s$, with V_0 a constant.

Within a good approximation, we can neglect the present contribution of density of baryons, neutrinos etc., $\rho_{om} \ll \rho_{o\Phi}$ because their contribution represents less than 5% of the matter of the Universe. The next step is to determine which is the scalar field potential. Fortunately a flat Universe dominated by scalar field with the function $F = V_0 a^s$ has a very important property. We can enunciate this property in the following theorem:

Theorem 1. *Let $\rho_\Phi = \frac{6}{a^6} \int \frac{F}{a} da$ with $F = V_0 a^s$ in a flat Universe dominated by a scalar field. Then the*

scalar field potential $V(\Phi)$, is essentially exponential in the regions where the scalar energy density dominates.

Proof: If the Universe is flat, $k = 0$. From equation (6) it follows that

$$\Phi = \sqrt{\frac{6-s}{\kappa_0}} \int \frac{da}{a} \sqrt{\frac{1}{1 + \left(\frac{\rho_m}{\rho_\Phi}\right)}}. \quad (11)$$

Thus, if the scalar field dominates ($\rho_m \ll \rho_\Phi$), this implies $a \simeq \exp(\sqrt{\frac{\kappa_0}{6-s}}\Phi)$. Then, it follows $V(\Phi) = F(a)/a^6 \simeq V_0 \exp(-\sqrt{\kappa_0(6-s)}\Phi)$. ■

This result strongly states that the scalar potential can only be exponential when the scalar field dominates with no other possibilities like “power-law” or “cosine”.

The theorem fulfills very well the present conditions of the Universe with the hypothesis we are investigating. Thus, we will take an exponential potential for the model of the Universe, which implies an extraordinary concordance with the scalar potential used to explain the rotation curves of galaxies [7].

With the conditions $C = 0$ and $F = V_0 a^s$, equations (5) and (4) are easily integrated for a flat Universe. One obtains [9,12]

$$a(t) = (K(t - t_0))^\lambda$$

$$\Phi - \Phi_0 = \sqrt{\frac{6-s}{\kappa_0}} \ln a$$

where $\lambda = 2/(6-s)$. The important quantities obtained from the solution in terms of the parameter λ are: the scalar field and the scalar potential

$$\Phi(a(t)) = \sqrt{\frac{2}{\kappa_0 \lambda}} \ln(a) \quad (12)$$

$$V(\Phi) = V_o \exp\left(-\sqrt{\frac{2\kappa_o}{\lambda}}\Phi\right), \quad (13)$$

the energy density of the scalar field

$$\rho_\Phi = \rho_{o\Phi} a^{-\frac{2}{\lambda}}$$

$$\rho_{o\Phi} = \frac{6V_o}{6-\frac{2}{\lambda}},$$

the state equation of the scalar field

$$w_\Phi = \frac{2}{3\lambda} - 1$$

where $p_\Phi = \frac{1}{2}\dot{\Phi}^2 - V(\Phi) = w_\Phi \rho_\Phi$. The scale factor

$$a(t) = \left(\frac{t}{t_o}\right)^\lambda,$$

where t_o is a normalization constant. The Hubble parameter

$$H = \frac{\dot{a}}{a} = \lambda t^{-1}$$

and the deceleration parameter

$$q = -\frac{\ddot{a}}{\dot{a}^2} a = -\frac{\lambda-1}{\lambda}.$$

According to the solution (12 - 13), the expression for the luminosity distance now reads

$$d_l(z; \lambda, V_o, H_o) = \frac{(1+z)\lambda}{H_o(1-\lambda)} \left[\frac{6V_o}{s\rho_c}\right]^{-\frac{1}{2}} \left[1 - (1+z)^{(1-\frac{1}{\lambda})}\right] \quad (14)$$

where $w_\Phi = 1 - s/3$ and we have rescaled $a_0 = 1$ today, for $\lambda \neq 1$. Fitting (14) with the data of Ia supernovae [2,14] we find $\lambda = 1.83$ and $V_o = 0.78\rho_c$ for $\rho_{o\Phi} \sim 0.95\rho_c$ where ρ_c is the critical density ($\rho_c = 0.92 \times 10^{-29} \text{gcm}^{-3}$) (see Fig. 1).

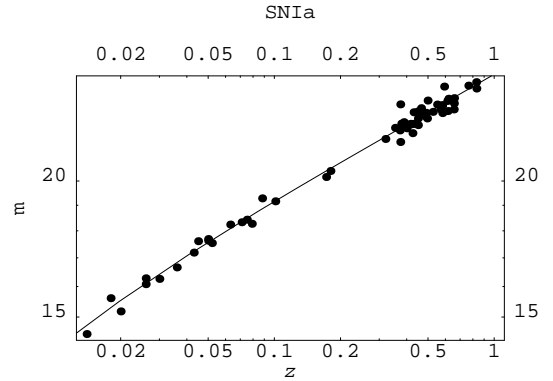


FIG. 1. Fit of the solution obtained for the value $\lambda = 1.83$. The dots represent the observational results and the solid line means $m(z) = \tilde{M} + 5 \log D_L$.

Now, we can calculate the deceleration parameter. We obtain $q_o = -0.45 = \text{constant}$, which really implies that the Universe is accelerating. For the density of the scalar field we obtain $\rho_\Phi = 0.95\rho_c a^{-1.09}$ and for its equation of state $w_\Phi = -0.636 = \text{constant}$. Currently, we are investigating the CMBR and the mass power spectrum. See [5] for a scalar field with equation of state $w = -2/3$ and Ω_Φ up to 0.8, where it is concluded that the scalar field fits all the required observations. If we use $H_o = 70 \frac{\text{Km}}{\text{sMpc}}$, we find:

$$t_o = 25.6 \times 10^9 \text{yr}.$$

t_o would be the age of a Universe that was always dominated by the scalar field, which is not our case.

The great concordance of our hypotheses with experimental results, suggests that the Universe lies at this moment in a scalar field dominated epoch. This permits us to speculate about the behavior of the Universe for red-shifts greater than $z = 1$ as restricted by SNIa observations. Observe that our results do not imply that

the Universe has been dominated by a scalar field during all its evolution. Instead, our model accepts the possibility of a Universe dominated by radiation or matter before the epoch we have analyzed. In order to draw a complete history of the Universe, we consider the periods of radiation and matter dominated eras. A general integration of the conservation equation for a perfect fluid made of radiation (dust), indicates that the density scales as $\rho_r = \rho_{or}a^{-4}$ ($\rho_m = \rho_{om}a^{-3}$), with $\rho_{or} = 10^{-5}\rho_c$ ($\rho_{om} = 0.05\rho_c$). In the FRW standard cosmology, the Universe was radiation dominated until $a \sim 10^{-3}$, the time when the density of radiation equals the density of matter. Recalling our result $\rho_\Phi = \rho_{o\Phi}a^{-1.09}$, the Universe changed to be matter dominated until $a \sim 0.21$, when the density of the scalar field equals the density of matter. At this time, the density of radiation is negligible. This corresponds to redshifts $z = 3.7$. The implications of this model are very strong. Since this time (approximately 14×10^9 yr. ago for this model), the scalar field began to dominate the expansion of the Universe and it enters in its actual acceleration phase, which includes most of the history of the Universe. Then we wonder if the scalar field is the responsible for the formation of structure too. According to [15], the formation of galaxies started at a few redshifts, from approximately 4.5 to 2, just when the scalar field began to be important.

Some final remarks. With our values, the solution is singular, i.e., $a(t)$ vanishes at some finite time. Moreover, the solution has no particle horizon [12] as can be seen from the expression (5) because $s > 4$. The question why nature uses only spin 1 and spin 2 fundamental interactions over the simplest spin 0 interactions becomes clear with our result. This result tells us that in fact nature has preferred the spin 0 interaction over the other two and in such case, the scalar field should thus be the responsible of the cosmos structure.

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